HISTORY OF TUNING AND TEMPERAMENT

Notes for the Spokane AGO Chapter

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Howard Stoess

I. The first step to understanding tuning and temperament is to take a look at the harmonic series.

A. A pure tone sounds only at the fundamental frequency or pitch.
B. Musical tones are complex in that they not only sound at the fundamental pitch, but also at higher frequencies sometimes called overtones. The first 5 harmonics of C are:

INTERVAL	RATIO	FACTOR
Octave	= 1:2	= 2.0X
Fifth	= 2:3	= 1.5X
Fourth	= 3:4	= 1.33X
Maj 3rd	= 4:5	= 1.25X
Min 3rd	= 5:6	= 1.2X

C. The first 5 harmonics, in cents:

INTERVAL	RATIO	CENTS
Octave	= 1:2	= 1200
Fifth	= 2:3	= 702
Fourth	= 3:4	= 498
Maj 3rd	= 4:5	= 386
Min 3rd	= 5:6	= 316

D. A circle of 12 pure 5ths = 8424 cents (12 X 702); 7 octaves = 8400 cents, (7 X 1200). This 24 cent discrepancy is known as the ditonic comma.

II. Equal Temperament

A. There is evidence that equal temperament was known in China as early as the 5th century, BC. It was introduced into western music early in the 16th century with the invention of fretted string instruments. It came into general use around 1854 in conjunction with the evolution of the modern piano and the introduction of chromaticism and impressionism in romantic music. Note that this date is 27 years after the death of Beethoven!
B. In theory, all semitones are expressed as the 12th root of 2, which like pi, is an irrational number. Several geometric and mathematical models have been used to reach approximate values for each note. The ratio of the semitone is slightly less than 18:17.
C. The equal tempered semitone is 100 cents, thus:

C	.	D	.	E	F	.	G	.	A	.	B	C
0	100	200	300	400	500	600	700	800	900	1000	1100	1200

D. One more look at the first 5 harmonics:

INTERVAL	PURE	EQUAL	DEVIATION
Octave	1200	1200	0
5^{th}	702	700	-2
4^{th}	498	500	+2
Maj 3^{rd}	386	400	-14
Min 3^{rd}	316	300	+16

E. Advantage of equal temperament:

1. You can play in any key. Every key is as in (or out) of tune as any other.

F. Disadvantages of equal temperament:

1. There is no key color.
2. There is a great amount of tension as maj 3rds are very wide and min 3rds are very narrow creating high beat rates (around 10/sec).
3. The high (and different) beat rates of the 3rds clash with the slow beat rate of the 5^{th} creating a very bad triad.

III. Historical tuning systems

A. Just
B. Meantone
C. Well, or irregular

IV. Just intonation

A. A family of temperaments based on pure octaves and pure 5ths - the first 2 harmonics

1. First described by the Greek philosopher and mathematician, Pythagoras, around 500 BC. Just intonation is often called Pythagorean tuning.
2. In its basic form, it is a line of pure (or just) 5ths beginning with F:

F – C – G – D – A – E – B

V. Meantone

A. Meantone tuning was developed during the Reformation.
B. Meantone is unique in that it is based on pure 3rds, rather than pure 5ths.
C. Meantone tuning is based on the syntonic comma.

1. The interval by which 4 pure 5ths exceed 2 octaves plus a pure maj 3rd. It is equal to 22 cents.

D. Characteristics of meantone

1. D is exactly midway between C and E - at the mean - and is why it is called meantone.
2. Ditonic semitones are larger than chromatic semitones (117 and 75 cents in ¼ comma meantone), therefore enharmonic equivalents do not exist. You must tune either a G# OR an Ab, for example, requiring retuning between pieces.
3. White keys cannot be accidentals and double flats and double sharps do not exist.
4. Because of the two previous items, it is classified as a restricted temperament.
5. The syntonic comma is distributed among the maj 3rds of the most remote keys, making them quite unusable with one or more very poor intervals called 'wolves'. These wolves can be camouflaged with the use of trills and other embellishments.
6. The best keys, C, F & G, also have pure, or nearly pure 4ths and 5ths giving them a very still, peaceful quality.
7. The character of the sound becomes more lively as sharps or flats are added.
8. The unevenness of the chromatic scale is quite apparent.

E. Comparison of Pietro Aron's ¼ syntonic comma meantone (c. 1523) to equal temperament:

C	C#	D	Eb	E	F	F#	G	G#	A	Bb	B	C
0	76	193	310	386	503	579	697	773	890	1007	1083	1200
0	−24	−7	+10	−14	+3	−21	−3	−27	−10	+7	−17	0

VI. Well, or irregular temperaments

A. Well temperament first appeared at about the same time as meantone, but didn't come into general use until the time of Bach and Handel. Recent scholarship has supported the proposition that this was the type of temperament that Bach himself preferred, not equal temperament as advanced by 19th century musicologists. His The Well Tempered Clavier was written to demonstrate that music could be played in all keys.
B. Characteristics of well temperaments

1. All keys and chords are usable.
2. C major and A minor are normally the best keys; very still like meantone. Movement increases as sharps or flats are added.
3. Most keys are better than equal temperament, with only the most remote keys slightly worse.
4. Keys have characteristic colors.

a. C, F, and G are very peaceful and serene and are often used for pastorales and other pieces of a quiet nature.
b. Keys with many sharps sound bright and cheerful and keys with many flats sound somber and dark. The composers made good use of key colors. Some temperaments have 3 or 4 groups of keys with similar colors, while other temperaments gradually change as flats or sharps are added.

5. There are no wolf intervals.
6. The comma is distributed in an irregular manner, rather than more evenly as in other systems.
7. Singers and instrumentalists have no problem adjusting to an irregular temperament, as the actual pitches used are very close to those of equal temperament.
8. The unevenness of the chromatic scale is not apparent.
9. Key modulation is available.
10. The raised keys are tempered to be enharmonic.

C. Irregular temperaments are modifications of one of the earlier tempering systems. Most are Pythagorean in that they are based on the ditonic comma and contain several pure 5ths.

D. Three temperaments in use today:

1. Werkmeister #1 - Andreas W. Werkmeister, 1691 (1/4 ditonic comma)
2. Kirnberger III - Johann Philip Kirnberger, 1779 (1/5 ditonic comma)

    a. Most Pythagorean with many pure 4ths and 5ths
    b. Very easy to tune

3. Young #2 - Thomas Young, 1800 (1/6 ditonic comma)
4. Comparison of Thomas Young's second temperament to equal temperament:

C	Db	D	Eb	E	F	Gb	G	Ab	A	Bb	B	C
0	90	196	294	392	498	588	698	792	894	996	1090	1200
0	−10	−4	−6	−8	−2	−12	−2	−8	−6	−4	−10	0

VII. References

A. Tuning the Historical Temperaments by Ear, Owen Jorgensen
B. Tuning and Temperament, J. Murray Barbour
C. Tuning Musical Instruments, John Meffen
D. Tuning and Temperament, John Brombaugh AGO Tape #M-2