HISTORY OF TUNING AND TEMPERAMENT

I. Why I became interested in tuning and temperament

A. What is the relationship between consecutive notes in the scale?

B. Why are the white keys and black keys arranged the way they are?

C. What is the difference between ‘tuning’ and ‘tempering’?

II. What do we mean when we say we are going to tune an instrument?

A. Most string, brass and wind players know how to tune their instruments, but very few can explain in detail what they are doing.

1. Explaining what it means to be ‘in tune’ is quite complex.

2. We are used to measuring systems where the intervals between larger units are logical and easy to visualize. EX:

   a. $4 \text{ C} + 3(1/3) \text{ C} = 5 \text{ C}$
   b. $7 \text{ LB} + 1/2 \text{ LB} = 8 \text{ LB} - 1/2 \text{ LB}$
   c. $1' + 7'' = 2' - 5''$

3. Unfortunately, our 12-tone chromatic scale doesn’t divide so neatly.

III. The first step to understanding this problem is to look at the harmonic series.

A. A pure tone sounds only at the fundamental frequency or pitch.

B. Musical tones are complex in that they not only sound at the fundamental pitch, but also at higher frequencies sometimes called overtones. The first 5 harmonics of C are:

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>RATIO</th>
<th>FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octave</td>
<td>1:2</td>
<td>2.0X</td>
</tr>
<tr>
<td>Fifth</td>
<td>2:3</td>
<td>1.5X</td>
</tr>
<tr>
<td>Fourth</td>
<td>3:4</td>
<td>1.33X</td>
</tr>
<tr>
<td>Maj 3rd</td>
<td>4:5</td>
<td>1.25X</td>
</tr>
<tr>
<td>Min 3rd</td>
<td>5:6</td>
<td>1.2X</td>
</tr>
</tbody>
</table>

C. It is the presence of these harmonics that give each musical instrument its peculiar sound. If we hear a trumpet, a clarinet and a violin play the same note, we identify the source of the note by the blending of the harmonics. The fundamental gives us the pitch, the harmonics identify the source. The same is true of the various voices of the pipe organ. The pipe builder causes certain harmonics to sound with the fundamental by the shape of the pipe, the material from which it is made and by using a particular scale or ratio (diameter to length). Mutations and mixtures sound at the exact harmonics of the fundamental, adding to the natural harmonics of the fundamental pitches.

D. These harmonics are what we listen for when we tune an instrument.

   1. For example, if we wish to tune the 5th C - G, the second harmonic of C and the first harmonic of G is G in the next higher octave. If our interval is not tuned pure, you can hear a beat produced by the out-of-tune harmonics. If it is pure, you will not hear a beat.
E. Let’s look at how some of these intervals work together:

1. A pure 5<sup>th</sup> + a pure 4<sup>th</sup> = 1 octave.
   
a. If C’ = 100, then 100 X 1 ½ = 150 (G’) X 1 1/3 = 200 (C”).

2. Trying to divide the octave in half or thirds doesn’t work so well.

3. For example, 3 maj 3rds do not = an octave.
   
a. 1.25 X 1.25 X 1.25 = 1.953125

F. To make division of the scale easier to understand, we use a logarithmic system that divides the scale into 1200 cents per octave.

G. Looking again at the first 5 harmonics:

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>RATIO</th>
<th>CENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octave</td>
<td>1:2</td>
<td>1200</td>
</tr>
<tr>
<td>Fifth</td>
<td>2:3</td>
<td>702</td>
</tr>
<tr>
<td>Fourth</td>
<td>3:4</td>
<td>498</td>
</tr>
<tr>
<td>Maj 3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>4:5</td>
<td>386</td>
</tr>
<tr>
<td>Min 3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>5:6</td>
<td>316</td>
</tr>
</tbody>
</table>

2. A pure 5<sup>th</sup> (702) + a pure 4<sup>th</sup> (498) = 1 octave (1200).

3. 3 major 3rds (386 X 3) = 1158 cents, 42 cents short of an octave, nearly half of a semitone flat!

H. Most of us have been told since very early in our musical training that notes with enharmonic names are the same pitch. In theory this is true only in equal temperament. For example:

1. Pure maj 3<sup>rd</sup> C - E = 386
   
   Pure maj 3<sup>rd</sup> E - G# = 386
   
   Pure dim 4<sup>th</sup> Ab - C = 428
   
   Octave = 1200

2. By changing the G# to an Ab we make the last interval a dim 4<sup>th</sup>, which completes the octave. The note G#/Ab either must be tuned and used as one or the other, or tempered to serve as both.

I. A circle of 12 pure 5ths = 8424 cents (12 X 702); 7 octaves = 8400 cents (7 X 1200). This 24 cent discrepancy is known as the ditonic comma.

J. The elimination or accommodation of the comma is called tempering. Tempering is an adjustment of the intervals between notes away from pure. It is a compromise to make the intervals fit.

K. It is common to use the term ‘tune’ for both tuning and tempering.

1. ‘Tuning’ is static and involves adjusting the pitch of one instrument to another, as when a group of instrumentalists ‘tune up’. It is also the correct term to use when a string player ‘tunes’ the individual strings of his instrument to each other.
   
a. When we ‘tune’, we adjust to unisons or pure intervals. These intervals can be expressed as the ratio of integers.

2. ‘Tempering’ is the correct term to use when we adjust pitches that are not pure, but set to some other value. This would apply to fretted string instruments and keyboard instruments.
   
a. In tempering, we distribute the comma over the 12 semitone intervals in the octave.

   b. These intervals can only be expressed with irrational numbers.
The remainder of this discussion assumes we are dealing with a keyboard instrument, specifically the organ.

1. The tempering of an organ is especially critical because:
   a. Except for the reeds, the tone is quite pure – free of complex harmonics.
   b. The volume of the tone does not decay, as on the piano.

IV. The solution to the problem - tempering the scale

A. There are approximately 150 tempering schemes that have been advanced over the centuries, with probably no more than 20 being practical for general use. Of these, about 10 are practical for keyboard instruments, and only one – equal temperament – practical for the modern piano.

V. Standard European 1/12 Diatonic Comma Equal Temperament

A. We will look at equal temperament first, even though it is considered a modern temperament.
   1. It is the most familiar.
   2. It is the least difficult to understand because it divides into 12 equal semitones.
   3. It serves as a good model with which to compare the earlier temperaments.

B. There is evidence that equal temperament was known in China as early as the 5th century, BC. It was introduced into western music early in the 16th century with the invention of fretted string instruments. It came into general use around 1854 in conjunction with the evolution of the modern piano and the introduction of chromaticism and impressionism in romantic music. Note that this date is 27 years after the death of Beethoven!

C. In equal temperament, every 5th is narrow by 2 cents, (702 - 2 = 700), and every 4th is wide by 2 cents, (498 + 2 - 500). This allows either circle to close.
   1. 12 X 700 = 7 X 1200 = 8400.
   2. 12 X 500 = 5 X 1200 = 6000.

D. In the middle C octave, these 2-cent deviations cause the 4ths and 5ths to beat at approximately one beat/sec.

E. In theory, all semitones are expressed as the 12th root of 2, which like pi, is an irrational number. Several geometric and mathematical models have been used to reach approximate values for each note. The ratio of the semitone is slightly less than 18:17.

F. The equal tempered semitone is 100 cents, thus:

   0   100  200  300  400  500  600  700  800  900 1000 1100 1200

G. One more look at the first 5 harmonics:

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>PURE</th>
<th>EQUAL</th>
<th>DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octave</td>
<td>1200</td>
<td>1200</td>
<td>0</td>
</tr>
<tr>
<td>5th</td>
<td>702</td>
<td>700</td>
<td>-2</td>
</tr>
<tr>
<td>4th</td>
<td>498</td>
<td>500</td>
<td>+2</td>
</tr>
<tr>
<td>Maj 3rd</td>
<td>386</td>
<td>400</td>
<td>+14</td>
</tr>
<tr>
<td>Min 3rd</td>
<td>316</td>
<td>300</td>
<td>-16</td>
</tr>
</tbody>
</table>

H. Enharmonic notes are the same pitch (C# and Db are the same note), and 3 equally tempered Maj 3rds DO equal an octave (3 X 400 = 1200).
I. Advantage of equal temperament:

1. You can play in any key. Every key is as in (or out) of tune as any other.

J. Disadvantages of equal temperament:

1. There is no key color?

2. There is a great amount of tension as maj 3rds are very wide and min 3rds are very narrow creating high beat rates (around 10/sec).

3. The high (and different) beat rates of the 3rds clash with the slow beat rate of the 5th creating a very bad triad.

K. Our first written instructions for setting equal temperament come from Giovanni Maria Lanfranco in 1533:

1. “The 5ths are tuned so flat that the ear is not well pleased with them; and the 3rds are as sharp as can be endured.”

L. Equal temperament has had its share of critics. Very few composers or organists preferred equal temperament until the French Romantic school.

1. In 1879, William Pole wrote in his book The Philosophy of Music, “The modern practice of tuning all organs to equal temperament has been a fearful detriment to their quality of tone. Under the old tuning, an organ made harmonious and attractive music. Now, the harsh 3rds give it a cacophonous and repulsive effect.”

2. In 1940, L. S. Lloyd wrote an article entitled The Myth of equal Temperament in which he described the improbability of singers, or players of any instrument with variable intonation of being able to sing or play in true equal temperament; or, a keyboard instrument actually being tuned to theoretically correct equal temperament.

M. Look at tuning scheme

VI. Historical tuning systems

A. Just

B. Meantone

C. Well, or irregular

VII. Just intonation

A. A family of temperaments based on pure octaves and pure 5ths – the first 2 harmonics

B. Used with pentatonic, diatonic and modal scales

C. Values for notes within the scale are mathematically derived.

1. First described by the Greek philosopher and mathematician, Pythagoras, around 500 BC. Just intonation is often called Pythagorean tuning.

D. This tuning is not suitable for keyboard instruments. It is important because most of the later systems are Pythagorean and it is the basis of our diatonic (12-tone) scale.

E. In its basic form, it is a line of pure (or just) 5ths beginning with F:

\[ F \rightarrow C \rightarrow G \rightarrow D \rightarrow A \rightarrow E \rightarrow B \]
F. Steps in the development of the Pythagorean scale, by 5ths and 4ths (inverted 5ths):

1. Establish first pure 5th and first note of bearing octave.
2. Create whole tone, F – G.
3. Create 2nd whole tone, C – D.
4. Create 3rd whole tone, G – A; and complete primitive pentatonic (5-tone) scale.
5. Create whole tone, D – E; and semitone, E – F.
6. Create whole tone, A – B; and semitone B – C. Completes diatonic scale of 5 whole tones and 2 semitones.
7. Create whole tone, Bb – C; and 2 semitones, A – Bb & Bb – B. It was raised because it creates 2 semitones, rather than 1. It would also have physically widened the octave which was mostly standardized by the Middle Ages. Also the width of the whole tone, A – B would have been half again as wide as the other whole tones.
8. Create whole tone, E – F#; and 2 semitones, F – F# & F# – G.
9. Create whole tone, Eb – F; and 2 semitones, D – Eb & Eb – E.
10. Create 2 whole tones, B – C# & C# – Eb; and 2 semitones, C – C# & C# – D.
11. Create 2 whole tones, F# – G# & G# – Bb; and 2 semitones, G – G# & G# – A. Leaves very bad wolf, Eb – G#.

G. Increased use of the chromatic scale, caused just intonation to become impractical.

VIII. Meantone

A. Meantone tuning was developed during the Reformation.

B. Meantone is unique in that it is based on pure 3rds, rather than pure 5ths.

C. We earlier described the ditonic or Pythagorean comma, which was based on a circle of 5ths and equals 24 cents. There is also a comma known as the syntonic comma.

1. The interval by which 4 pure 5ths exceed 2 octaves plus a pure maj 3rd. It is equal to 22 cents. This is the comma used in most meantone temperaments.

D. Characteristics of meantone

1. D is exactly midway between C and E – at the mean – and is why it is called meantone.
2. Ditonic semitones are larger than chromatic semitones (117 and 75 cents in ¼ comma meantone), therefore enharmonic equivalents do not exist. You must tune either a G# OR an Ab, for example, requiring re-tuning between pieces.
3. White keys cannot be accidentals and double flats and double sharps do not exist.
4. Because of the two previous items, it is classified as a restricted temperament.
5. The syntonic comma is distributed among the maj 3rds of the most remote keys, making them quite unusable with one or more very poor intervals called ‘wolves’. These wolves can be camouflaged with the use of trills and other embellishments.
6. The best keys, C, F & G, also have pure, or nearly pure 4ths and 5ths giving them a very still, peaceful quality.
7. The character of the sound becomes more lively as sharps or flats are added.
8. The unevenness of the chromatic scale is quite apparent.

E. Comparison of Pietro Aron’s ¼ syntonic comma meantone (c. 1523) to equal temperament:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>C#</th>
<th>D</th>
<th>Eb</th>
<th>E</th>
<th>F</th>
<th>F#</th>
<th>G</th>
<th>G#</th>
<th>A</th>
<th>Bb</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>76</td>
<td>193</td>
<td>310</td>
<td>386</td>
<td>503</td>
<td>579</td>
<td>697</td>
<td>773</td>
<td>890</td>
<td>1007</td>
<td>1083</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-24</td>
<td>-7</td>
<td>+10</td>
<td>-14</td>
<td>+3</td>
<td>-21</td>
<td>-3</td>
<td>-27</td>
<td>-10</td>
<td>+7</td>
<td>-17</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

F. Look at tuning scheme

IX. Well, or irregular temperaments

A. Earlier we discussed familiar measuring systems. There is one very common measurement that we temper in an irregular manner - the calendar. (discuss a 4-year period on the calendar – variable length of months and the leap year)

B. Just intonation, meantone and equal temperament are classified as regular systems because all, or all but one of the 5ths, are tempered equally.
C. Well temperament first appeared at about the same time as meantone, but didn’t come into general use until the time of Bach and Handel. Recent scholarship has supported the proposition that this was the type of temperament that Bach himself preferred, not equal temperament as advanced by 19th century musicologists. His *The Well Tempered Clavier* was written to demonstrate that music could be played in all keys.

D. Why well-tempering replaced meantone

1. Pre-Bach composers wrote to stay within the limits of meantone. They also used its characteristics to their advantage by favoring harmonies made up of 3rds and 6ths and using the poorer chords for contrast and affect. They also wrote in modal keys to avoid the wolf intervals.

2. Composers could write chromatic music, but the performer had to tune the raised keys to the correct intervals for the composition. This was common in music written for the harpsichord, but music written for the organ rarely exceeded 12 scale degrees – the raised keys always had the same value.

3. One attempt at overcoming the limitations of meantone was to split the raised keys with one part of the key playing an F# and the other part a Gb, for example. It never gained acceptance because of increased difficulty in construction, not to mention performance.

4. As Bach and others began writing music with more than 12 scale degrees, meantone was no longer practical. Many of Bach’s organ works contain 13 to 15 scale degrees and some go as high as 21. His entire keyboard output uses 25 scale degrees, including a double Eb and a double C#. Clearly, he had access to an instrument tuned to a newer temperament than meantone.

E. Characteristics of well temperaments

1. All keys and chords are usable.

2. C major and A minor are normally the best keys; very still like meantone. Movement increases as sharps or flats are added.

3. Most keys are better than equal temperament, with only the most remote keys slightly worse.

4. Keys have characteristic colors.
   a. C, F, and G are very peaceful and serene and are often used for pastorales and other pieces of a quiet nature.
   b. Keys with many sharps sound bright and cheerful and keys with many flats sound somber and dark. The composers made good use of key colors. Some temperaments have 3 or 4 groups of keys with similar colors, while other temperaments gradually change as flats or sharps are added.

5. There are no wolf intervals.

6. The comma is distributed in an irregular manner, rather than more evenly as in other systems.

7. Singers and instrumentalists have no problem adjusting to an irregular temperament, as the actual pitches used are very close to those of equal temperament.

8. The unevenness of the chromatic scale is not apparent.

9. Key modulation is available.

10. The raised keys are tempered to be enharmonic.

F. Irregular temperaments are modifications of one of the earlier tempering systems. Most are Pythagorean in that they are based on the ditonic comma and there are several pure 5ths.

G. Three temperaments in use today:

1. Werkmeister #1 - Andreas W. Werkmeister, 1691 (1/4 ditonic comma)
2. Kirnberger III - Johann Philip Kirnberger, 1779 (1/5 ditonic comma)
   a. Most Pythagorean with many pure 4ths and 5ths
   b. Very easy to tune
   c. Look at tuning scheme

3. Young #2 - Thomas Young, 1800 (1/6 ditonic comma)

4. Comparison of Thomas Young’s second temperament to equal temperament:

\[
\begin{array}{cccccccccccc}
\text{C} & \text{Db} & \text{D} & \text{Eb} & \text{E} & \text{F} & \text{Gb} & \text{G} & \text{Ab} & \text{A} & \text{Bb} & \text{B} & \text{C} \\
0 & 90 & 196 & 294 & 392 & 498 & 588 & 698 & 792 & 894 & 996 & 1090 & 1200 \\
0 & -10 & -4 & -6 & -8 & -2 & -12 & -2 & -8 & -6 & -4 & -10 & 0
\end{array}
\]

5. Look at tuning scheme

X. Practical thoughts about tuning and temperament

A. All music sounds best when played in the temperament that the composer was using at the time.

B. Tempering can be static, or change from moment to moment.

C. 3 categories of instruments:

1. Those capable of playing any pitch and not confined to the 12 tone system - tempering is continuous:
   a. Violin family
   b. Trombone
   c. Human voice

2. Those whose construction provides mechanical or acoustic means for producing the notes of the 12 tone system, but allow for minor changes in pitch by the player. The instrument is tuned before performance and tempered during performance:
   a. Fretted string instruments
   b. Valved brass instruments
   c. Woodwinds

3. Those whose construction allows for pre-performance tuning/tempering, sometimes by a technician, and no tempering during performance:
   a. Organ
   b. Piano
   c. Harpsichord
   d. Harp

D. The orchestra is a mix of temperaments.

1. Harp, tunable percussion instruments, piano and organ - usually equal temperament.

2. Most other instruments are just in nature and play flats slightly lower than the enharmonic sharps.

E. The voice

1. Singers will easily adjust to the temperament of any instrument with which they are singing.

2. An a cappella choir doesn’t sing in any recognized temperament. Singers, more so than string and woodwind players, constantly listen and make minute adjustments in pitch to stay in tune with each other by singing all principal intervals pure.
3. This constant tempering contributes to the tendency for unaccompanied singers to go flat.

4. This is not so likely to occur if the music is written in a modal key, or if the only chords are I, IV and V. It is very likely if there is modulation and/or chromatic passages.

E. Meantone is best suited for the performance of early music on the harpsichord or organ.

F. Equal temperament is the only practical temperament for the modern piano.

G. Large organs with romantic voicing and extensive unification should be tuned to equal temperament.

H. Most other organs, especially those with baroque voicing, should be tuned to a well temperament.

XI. References

A. Tuning the Historical Temperaments by Ear, Owen Jorgensen
B. Tuning and Temperament, J. Murray Barbour
C. Tuning Musical Instruments, John Meffen
D. Tuning and Temperament, John Brombaugh AGO Tape #M-2

XII. Table of intervals

<table>
<thead>
<tr>
<th>Interval</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>syntonic comma</td>
</tr>
<tr>
<td>24</td>
<td>ditonic comma</td>
</tr>
<tr>
<td>100</td>
<td>equally tempered semitone</td>
</tr>
<tr>
<td>112</td>
<td>diatonic pure semitone</td>
</tr>
<tr>
<td>200</td>
<td>equally tempered whole tone</td>
</tr>
<tr>
<td>204</td>
<td>pure whole tone</td>
</tr>
<tr>
<td>300</td>
<td>equally tempered min 3rd</td>
</tr>
<tr>
<td>316</td>
<td>pure min 3rd</td>
</tr>
<tr>
<td>386</td>
<td>pure maj 3rd</td>
</tr>
<tr>
<td>400</td>
<td>equally tempered maj 3rd</td>
</tr>
<tr>
<td>498</td>
<td>pure 4th</td>
</tr>
<tr>
<td>500</td>
<td>equally tempered 4th</td>
</tr>
<tr>
<td>600</td>
<td>equal tempered aug 4th/dim 5th</td>
</tr>
<tr>
<td>610</td>
<td>pure dim 5th</td>
</tr>
<tr>
<td>700</td>
<td>equally tempered 5th</td>
</tr>
<tr>
<td>702</td>
<td>pure 5th</td>
</tr>
<tr>
<td>800</td>
<td>equally tempered aug 5th/min 6th</td>
</tr>
<tr>
<td>814</td>
<td>pure min 6th</td>
</tr>
<tr>
<td>884</td>
<td>pure maj 6th</td>
</tr>
<tr>
<td>900</td>
<td>equally tempered maj 6th</td>
</tr>
<tr>
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</tr>
<tr>
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<td>octave</td>
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